# Luminosity measurements for the *R* scan experiment at BESIII

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**Abstract:** By analyzing the large-angle Bhabha scattering events $e^+e^- \to (\gamma)e^+e^-$ and diphoton events $e^+e^- \to (\gamma)\gamma\gamma$ for the data sets collected at center-of-mass (c.m.) energies between 2.2324 and 4.5900 GeV (131 energy points in total) with the upgraded Beijing Spectrometer (BESIII) at the Beijing Electron-Positron Collider (BEPCII), the integrated luminosities have been measured at the different c.m. energies, individually. The results are important inputs for the $R$ value and J/ψ resonance parameter measurements.

**Keywords:** luminosity, Bhabha, diphoton, $R$ value

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## 1 Introduction

Hadron production in $e^+e^-$ annihilation is one of the most valuable testing grounds for Quantum Chromodynamics (QCD), and is an important input for precision tests of the Standard Model (SM). The $R$ value, which is defined as the lowest-level hadronic cross section normal-

ized to the theoretical $\mu^+\mu^-$ production cross section in $e^+e^-$ annihilation, is an indispensable input for the determination of the non-perturbative hadronic contribution to the electromagnetic coupling constant evaluated at the Z pole ($\alpha(M_Z^2)$) [1, 2], and the anomalous magnetic moment $a_\mu = (g-2)/2$ of the muon [3]. The dominant uncertainties in both $\alpha(M_Z^2)$ and $a_\mu$ measurements are due to the effects of hadronic vacuum polarization, which cannot be reliably calculated in the low energy region. Instead, with the application of dispersion relations, experimentally measured $R$ values can determine the effect of vacuum polarization.

Experimentally, the $R$ value is determined from

$$R = \frac{N_{\rm had}^{\rm obs} - N_{\rm had}^{\rm bkg}}{\sigma_{\mu\mu}^0 \cdot \mathcal{L} \cdot \varepsilon_{\rm had} \cdot \varepsilon_{\rm had}^{\rm trig} \cdot (1+\delta)}, \quad (1)$$

where $N_{\rm had}^{\rm obs}$ is the number of observed hadronic events, $N_{\rm had}^{\rm bkg}$ is the number of background events, $\mathcal{L}$ is the integrated luminosity, $\varepsilon_{\rm had}$ is the detection efficiency for the hadron event selection, $\varepsilon_{\rm had}^{\rm trig}$ is the trigger efficiency, $1+\delta$ is the initial-state radiation (ISR) correction factor, and $\sigma_{\mu\mu}^0$ is the Born cross section of $e^+e^- \to \mu^+\mu^-$. Therefore, the measurement of integrated luminosity plays an important role in the $R$ value measurement.

Quantum electrodynamics (QED) processes are usually applied to determine the integrated luminosity, due to larger production rates, simpler final-state topologies and more accurate cross section calculation in theory relative to the other processes. The integrated luminosity is determined from

$$\mathcal{L} = \frac{N_{\rm QED}^{\rm obs} - N_{\rm QED}^{\rm bkg}}{\sigma_{\rm QED} \cdot \varepsilon_{\rm QED} \cdot \varepsilon_{\rm QED}^{\rm trig}}, \quad (2)$$

where $N_{\rm QED}^{\rm obs}$ is the number of QED events observed in the experimental data, $N_{\rm QED}^{\rm bkg}$ is the number of background events, $\sigma_{\rm QED}$ is the cross section of the selected QED process, $\varepsilon_{\rm QED}$ is the detection efficiency and $\varepsilon_{\rm QED}^{\rm trig}$ is the trigger efficiency.

In this paper, we present the measurements of luminosities of the $R$ scan data samples taken at BESIII from 2012 to 2014. The measurements are performed by analyzing two QED processes, $e^+e^- \to (\gamma)e^+e^-$ and $e^+e^- \to (\gamma)\gamma\gamma$. For energy points near the $J/\psi$ resonance, only the $e^+e^- \to (\gamma)\gamma\gamma$ process is used, because the Monte Carlo (MC) simulation at the $J/\psi$ resonance is sensitive to the c.m. energy and is imperfect.

## 2 Detector

BEPCII [4] is a double-ring $e^+e^-$ collider designed to provide a peak luminosity of $10^{33}$ cm$^{-2}\cdot$s$^{-1}$ at a c.m. energy ($\sqrt{s}$) of 3770 MeV. The BESIII [4] detector has a geometrical acceptance of 93% of $4\pi$ and has four main detector sub-components as follows. (1) A small-cell, helium-based (60% He, 40% $C_3H_8$) main drift chamber (MDC) with 43 layers providing an average single-hit resolution of 135 μm, and charged-particle momentum resolution in a 1 T magnetic field of 0.5% at 1 GeV/$c$. (2) An electromagnetic calorimeter (EMC) consisting of 6240 CsI(Tl) crystals in a cylindrical structure arranged in a barrel and two endcaps. The energy resolution at 1.0 GeV is 2.5% (5%) in the barrel (endcaps), and the position resolution is 6 mm (9 mm) in the barrel (endcaps). (3) A time-of-flight (TOF) system for particle identification composed of a barrel part made of two layers with 88 pieces of 5 cm thick, 2.4 m long plastic scintillator in each layer, and two endcaps with 96 fan-shaped, 5 cm thick, plastic scintillators in each endcap. The time resolution of 80 ps (110 ps) for the barrel (endcap) provides $2\sigma$ K/π separation for momenta up to $\sim$1.0 GeV/$c$. (4) A muon system (MUC) consisting of 1000 m$^2$ of resistive plate chambers in nine (eight) layers of barrel (endcap) provides 2 cm position resolution.

## 3 Data sample and Monte Carlo simulation

The measurements of luminosities were performed for 131 data samples, including 4 energy points at 2.2324, 2.4000, 2.8000, 3.4000 GeV taken at the 2012 run, 104 energy points from 3.8500 to 4.5900 GeV taken at the 2013–2014 runs, 15 energy points near the $J/\psi$ production threshold, 4 energy points during the τ mass measurement and 4 energy points for charmonium studies.

The $e^+e^- \to (\gamma)e^+e^-$, $(\gamma)\gamma\gamma$ and $(\gamma)\mu^+\mu^-$ events were simulated with the generator Babayaga v3.5 [5–7]. The background process of $e^+e^- \to \tau^+\tau^-$ was generated with the KKMC [8], while the $e^+e^- \to$ hadrons and $e^+e^- \to e^+e^-$ + X (X can be hadrons or leptons) events were generated with LUARLW [9] and BesTwogam [10], respectively.

## 4 Analysis

The $e^+e^- \to (\gamma)e^+e^-$ events are required to have two good charged tracks with opposite charge. Each charged track is required to be within ±10 cm of the interaction point in the beam direction and 1 cm in the plane perpendicular to the beam. In addition, the charged tracks are required to be within $|\cos\theta| < 0.8$, where $\theta$ is the polar angle, in the MDC. Without applying further particle identification, the tracks are assigned as electron and positron depending on their charges. The deposited energies of electron and positron ($E_{e^\pm}$) in the EMC are required to be larger than $0.65 \times E_{\rm beam}$ to suppress backgrounds, where $E_{\rm beam}$ is the beam energy. To make sure

the selected charged tracks are back to back in the c.m. system, $|\Delta\theta_{e^\pm}| = |\theta_1+\theta_2-180^\circ| < 10.0^\circ$ and $|\Delta\phi_{e^\pm}| = ||\phi_1-\phi_2|-180^\circ| < 5.0^\circ$ are required, where $\theta_{1/2}$ and $\phi_{1/2}$ are the polar and azimuthal angles of the two charged tracks, respectively. Figure 1 shows comparisons of the momentum and polar angle distributions of electrons and positrons between experimental data and MC simulation at $\sqrt{s} = 2.2324$ GeV. Good agreement is observed.

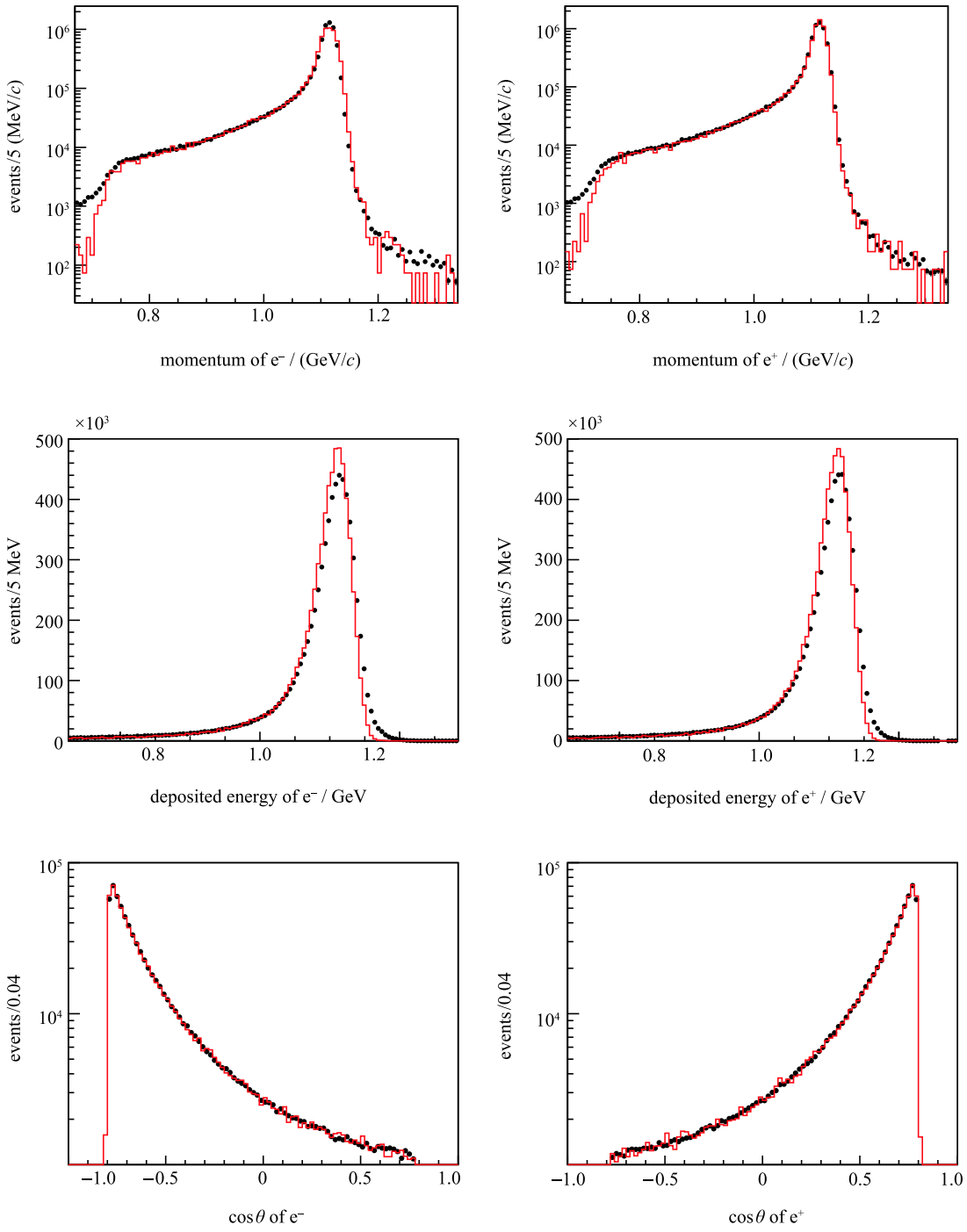


Fig. 1. (color online) The distributions of momentum (upper plots), deposited energy (middle plots) and polar angle $\cos\theta$ (lower plots) for electrons (left) and positrons (right) at $\sqrt{s} = 2.2324$ GeV. Dots with error bars are experimental data and red histograms are signal MC simulation. The MC entries are normalized to the experimental data.

To select $e^+e^- \to (\gamma)\gamma\gamma$ events, the number of good charged tracks is required to be zero. Two neutral clusters are required to have a polar angle $|\cos\theta| < 0.8$ with the deposited energy $E_\gamma$ satisfying $0.7 < E_\gamma/E_{\rm beam} < 1.16$. The two selected photon candidates are further required to be back to back by applying the requirement $|\Delta\phi_\gamma| = |\phi_{\gamma 1} - \phi_{\gamma 2}| < 2.5^\circ$, where $\phi_{\gamma 1/2}$ are the azimuthal anlge of the photons. Figure 2 shows comparisons of the energy deposition, polar angle and $\Delta\phi_\gamma$ distributions of two selected photons between experimental data and MC simulation at $\sqrt{s} = 2.2324$ GeV.

The numbers of observed QED events, $N^{\rm obs}_{\rm QED}$, are obtained by event counting after applying the event selection requirements to experimental data at different c.m. energies, individually. The detection efficiencies of signals, $\varepsilon_{\rm QED}$, are obtained by analyzing the corresponding signal MC events as done in data analysis. The cross sections of selected QED processes are calculated with the Babayaga v3.5 generator and the trigger efficiencies are quoted from Ref. [11].

To estimate the numbers of background events, $N^{\rm bkg}_{\rm QED}$, two different methods are applied for $e^+e^- \to (\gamma)e^+e^-$ and $e^+e^- \to (\gamma)\gamma\gamma$ processes, individually. For the $e^+e^- \to (\gamma)e^+e^-$ process, the numbers of background events are estimated by performing the same requirements on the background MC samples, which yields a background level of $10^{-5}$ after normalization. For $e^+e^- \to (\gamma)\gamma\gamma$ process, the background level is relatively large due to the hadronic process contamination. The normalized numbers of background events from $e^+e^- \to (\gamma)\gamma\gamma$ are estimated from the $\Delta\phi_\gamma$ sideband region, defined as $2.5^\circ < |\Delta\phi_\gamma| < 5.0^\circ$. The distributions of the $\Delta\phi_\gamma$ sideband is supposed to be flat by analyzing the background MC samples.

Table 1 shows the input numbers used to calculate the luminosities at $\sqrt{s} = 2.2324$ and 3.0969 GeV.

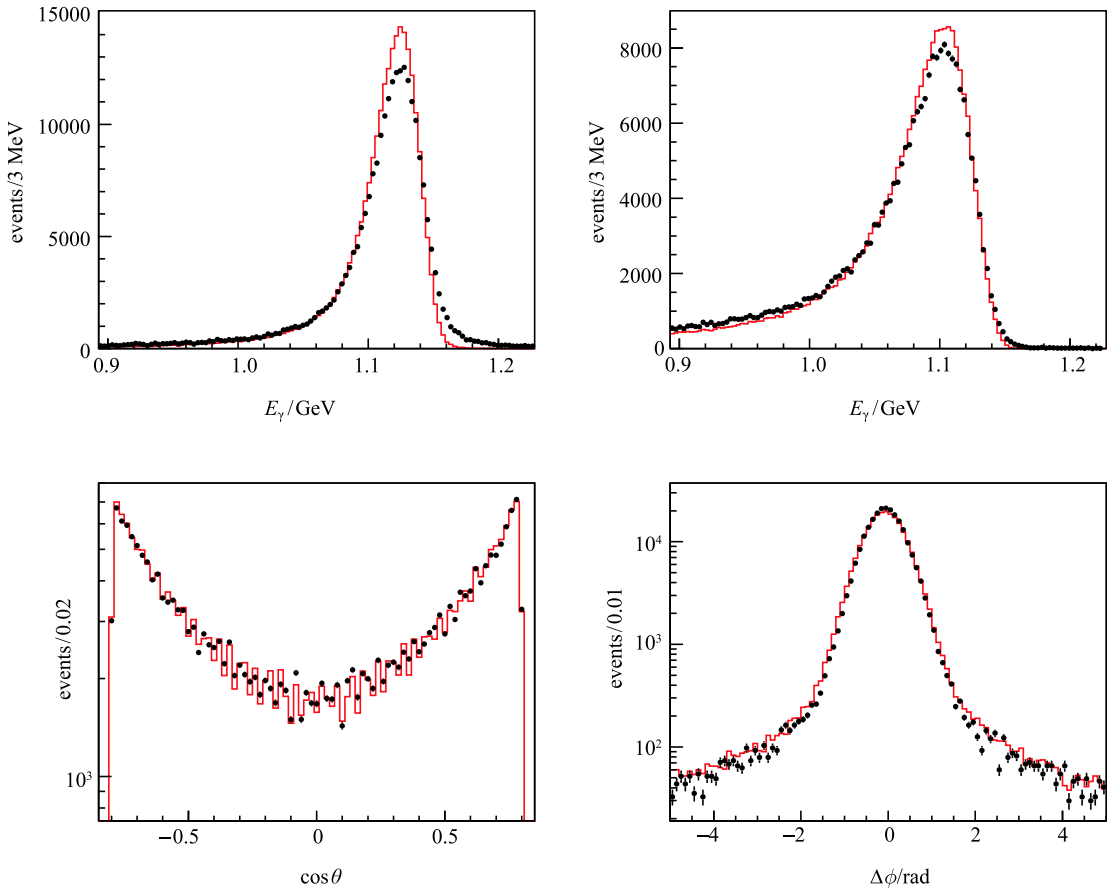


Fig. 2. (color online) Deposited energy distributions of the most energetic $\gamma$ (upper left), the second most energetic $\gamma$ (upper right), cos$\theta$ (bottom left) and $\Delta\phi$ (bottom right) at $\sqrt{s} = 2.2324$ GeV. Dots with error bars are experimental data and red histograms are signal MC simulation. The MC entries are normalized to the experimental data. The discrepancies in the deposited energy distributions are due to the imperfect simulation of energy correction deposited in the TOF. However, it will not affect the efficiency, since loose requirements on these variables are applied. The uneven distribution of cos$\theta$ is due to the structure of the crystals in the EMC.

Table 1. Summaries of the input numbers in luminosity calculation at $\sqrt{s}$ = 2.2324 and 3.0969 GeV.

| $\sqrt{s}$/GeV | QED process | $N^{\rm obs}_{\rm QED}$ | $N^{\rm bkg}_{\rm QED}$ | $\sigma_{\rm QED}$ /nb | $\varepsilon_{\rm QED}$(%) | $\varepsilon^{\rm trig}_{\rm QED}$(%) | $\mathcal{L}$/pb$^{-1}$ |
|---|---|---|---|---|---|---|---|
| 2.2324 | $(\gamma)e^+e^-$ | 728522 | 8 | 1476.5 | 18.74 | 100 | 2.645 |
| 2.2324 | $(\gamma)\gamma\gamma$ | 86974 | 1138 | 70.26 | 46.50 | 100 | 2.627 |
| 3.0969 | $(\gamma)\gamma\gamma$ | 36083 | 1062 | 36.59 | 46.25 | 100 | 2.069 |

## 5 Systematic uncertainty

The main systematic uncertainties of the integrated luminosity originate from the uncertainties related to the requirements on the kinematic variables, tracking efficiency, cluster reconstruction efficiency, c.m. energy, MC statistics, background estimation, trigger efficiency and generators.

For the systematic uncertainty from requirements on each kinematic variable, we re-measure the luminosity by altering the required values, i.e., $|\cos\theta| < 0.8$ changes to $|\cos\theta| < 0.75$, $|\Delta\theta_{e^\pm}| < 10^\circ$ changes to $|\Delta\theta_{e^\pm}| < 15^\circ$, $|\Delta\phi_{e^\pm}| < 5^\circ$ changes to $|\Delta\phi_{e^\pm}| < 10^\circ$, $|\Delta\phi_\gamma| < 2.5^\circ$ changes to $|\Delta\phi_\gamma| < 3.0^\circ$, $E_{e^\pm}/E_{\rm beam} > 0.65$ changes to $E_{e^\pm}/E_{\rm beam} > 0.7$ and $0.7 < E_\gamma/E_{\rm beam} < 1.16$ changes to $0.74 < E_\gamma/E_{\rm beam} < 1.2$, individually. The resultant differences of measured luminosity with respect to the nominal value are taken as the systematic uncertainty.

To study the uncertainty of tracking efficiency, a Bhabha event sample is selected with only EMC information [12]. The candidate events are selected by requiring the two clusters detected in the EMC with the deposited energy larger than $0.65 \times E_{\rm beam}$ and having the polar angle $|\cos\theta| < 0.8$, corresponding to the angular coverage of the barrel EMC. The two shower clusters in the $xy$-plane of the EMC are not back to back, since the two clusters originating from $e^\pm$ in the $e^+e^- \to (\gamma)e^+e^-$ candidate events are bent in the magnetic field. $\Delta\phi_{e^\pm}$ is required to be in the range of $[-40^\circ, -5^\circ]$ or $[5^\circ, 40^\circ]$ to remove the $e^+e^- \to (\gamma)\gamma\gamma$ events. We further apply the MDC information on the selected candidates, and the ratio of surviving events is regarded as the tracking efficiency. The average difference in the tracking efficiency between data and signal MC simulation, 0.41%, is taken as the systematic uncertainty.

The systematic uncertainty due to the cluster reconstruction efficiency in the EMC is determined to be 0.05% for $e^\pm$ by comparing the cluster reconstruction efficiencies between data and signal MC (both for $e^+$ and $e^-$). Since high-energy $\gamma$ and $e^\pm$ have similar behaviour in the EMC, the value of 0.05% is also taken as the systematic uncertainty due to the cluster reconstruction efficiency in the EMC for a single $\gamma$.

The uncertainty of c.m. energy is estimated to be 2 MeV [13]. For each energy point, an alternative MC simulation sample of 1 million events with a c.m. energy of 2 MeV above the nominal value was generated to re-estimate the detection efficiency, and the difference in the results is regarded as the systematic uncertainty due to c.m. energy.

The uncertainty of MC statistics is 0.17% for the $e^+e^- \to (\gamma)e^+e^-$ process and 0.15% for the $e^+e^- \to (\gamma)\gamma\gamma$ process, which is estimated by

$$\frac{1}{\sqrt{N}} \cdot \sqrt{\frac{(1-\varepsilon)}{\varepsilon}}, \tag{3}$$

where $N$ is the number of signal MC events, and $\varepsilon$ is the detection efficiency.

The rate of background events in the selected $e^+e^- \to (\gamma)e^+e^-$ candidate events is very small ($10^{-5}$). Therefore, the uncertainty due to background contamination is neglected. For $e^+e^- \to (\gamma)\gamma\gamma$ events, the rate of background events is the normalized number of selected background events in the sideband region divided by the number of signal events, which are (1.53±0.03)% and (1.31±0.04)% for experimental data and the MC simulation, respectively. Therefore, the difference 0.23% is taken as the uncertainty from background contamination.

The trigger efficiencies for barrel $e^+e^- \to (\gamma)e^+e^-$ events and $e^+e^- \to (\gamma)\gamma\gamma$ events are 100% with an uncertainty of less than 0.1% [11].

The uncertainty due to the Babayaga generator v3.5 is 0.5% for $e^+e^- \to (\gamma)e^+e^-$, while 1.0% for $e^+e^- \to (\gamma)\gamma\gamma$ [6].

Table 2. Summary of systematic uncertainties at $\sqrt{s}$ = 2.2324 GeV.

| source | $e^+e^- \to (\gamma)e^+e^-$ | $e^+e^- \to (\gamma)\gamma\gamma$ |
|---|---|---|
| $\lvert\cos\theta\rvert < 0.8$ | 0.12 | 0.18 |
| $\lvert\Delta\theta_{e^\pm}\rvert < 10^\circ$ | 0.05 | — |
| $\lvert\Delta\phi_{e^\pm}\rvert < 5^\circ$ | 0.01 | — |
| $\lvert\Delta\phi_\gamma\rvert < 2.5^\circ$ | — | 0.07 |
| $E_{e^+}/E_{\rm beam} > 0.65$ | 0.04 | — |
| $E_{e^-}/E_{\rm beam} > 0.65$ | 0.05 | — |
| $0.7 < E_\gamma/E_{\rm beam} < 1.16$ | — | 0.10 |
| tracking efficiency | 0.41 | — |
| cluster reconstruction | 0.10 | 0.10 |
| beam energy | 0.09 | 0.09 |
| MC statistics | 0.17 | 0.15 |
| background estimation | 0.00 | 0.23 |
| trigger efficiency | 0.10 | 0.10 |
| generator | 0.50 | 1.00 |
| total | 0.70 | 1.10 |

Systematic uncertainties at $\sqrt{s}$ = 2.2324 GeV for $e^+e^- \to (\gamma)e^+e^-$ and $e^+e^- \to (\gamma)\gamma\gamma$ are listed in Table 2. Assuming all sources of systematic uncertainties are uncorrelated, the total uncertainty is calculated to be 0.7% for $e^+e^- \to (\gamma)e^+e^-$ and 1.1% for $e^+e^- \to (\gamma)\gamma\gamma$ by adding all the contributions in quadrature. The uncertainties related to the tracking efficiency, cluster reconstruction efficiency, trigger efficiency and generators are common between the different c.m. energy points, while others are c.m. energy dependent and are determined for the different c.m. energy points, individually.

## 6 Summary

By using the QED processes $e^+e^- \to (\gamma)e^+e^-$ and $e^+e^- \to (\gamma)\gamma\gamma$, the integrated luminosities have been measured for 131 data samples with c.m. energy between 2.2324 and 4.5900 GeV. The precision of integrated luminosity is around 0.7% for $e^+e^- \to (\gamma)e^+e^-$, and around 1.1% for $e^+e^- \to (\gamma)\gamma\gamma$. The total luminosity is 1036.3 $pb^{-1}$, and the luminosities at the individual c.m. energy points are summarized in Table3. The ratios of the measured luminosities from the two process are illustrated in Fig. 3. The ratios are close to 1 within the uncertainties, which indicates the results from the two measurements are consistent with each other. For each energy point out of the J/ψ resonance region, the luminosity measured by $e^+e^- \to (\gamma)e^+e^-$ is more precise and thus is recommended. For energy points around J/ψ (from 3.0930 to 3.1200 GeV), only the luminosities measured by $e^+e^- \to (\gamma)\gamma\gamma$ are obtained. The measured results are important inputs for physics studies, *e.g.*, the $R$ value measurement and J/ψ resonance parameter measurement.

Table 3. The summaries of measured integrated luminosities from the two QED processes. The first uncertainty is statistical and the second is systematic.

| $\sqrt{s}$/GeV | $e^+e^- \to (\gamma)e^+e^-$/$pb^{-1}$ | $e^+e^- \to (\gamma)\gamma\gamma$/$pb^{-1}$ |
|---|---|---|
| 2.2324 | 2.645±0.006±0.020 | 2.627±0.009±0.028 |
| 2.4000 | 3.415±0.007±0.024 | 3.428±0.011±0.040 |
| 2.8000 | 3.753±0.008±0.026 | 3.766±0.014±0.042 |
| 3.0500 | 14.893±0.030±0.103 | 14.919±0.029±0.158 |
| 3.0600 | 15.040±0.030±0.131 | 15.060±0.029±0.158 |
| 3.0800 | 31.019±0.060±0.189 | 30.942±0.044±0.338 |
| 3.0830 | 4.740±0.011±0.029 | 4.769±0.017±0.052 |
| 3.0900 | 15.709±0.031±0.099 | 15.558±0.030±0.162 |
| 3.0930 | — | 14.910±0.030±0.157 |
| 3.0943 | — | 2.143±0.011±0.023 |
| 3.0952 | — | 1.816±0.010±0.019 |
| 3.0958 | — | 2.135±0.011±0.023 |
| 3.0969 | — | 2.069±0.011±0.024 |
| 3.0982 | — | 2.203±0.011±0.023 |
| 3.0990 | — | 0.756±0.007±0.008 |
| 3.1015 | — | 1.612±0.010±0.018 |
| 3.1055 | — | 2.106±0.011±0.022 |
| 3.1120 | — | 1.720±0.010±0.019 |
| 3.1200 | — | 1.264±0.009±0.013 |
| 3.4000 | 1.733±0.005±0.014 | 1.754±0.012±0.020 |
| 3.5000 | 3.633±0.009±0.025 | 3.643±0.017±0.040 |
| 3.5424 | 8.693±0.019±0.060 | 8.711±0.027±0.098 |
| 3.5538 | 5.562±0.013±0.034 | 5.593±0.021±0.059 |
| 3.5611 | 3.847±0.009±0.028 | 3.894±0.018±0.043 |
| 3.6002 | 9.502±0.020±0.076 | 9.620±0.028±0.108 |
| 3.6500 | 48.385±0.094±0.300 | 48.618±0.065±0.538 |
| 3.6710 | 4.628±0.011±0.028 | 4.603±0.020±0.052 |
| 3.8500 | 7.967±0.018±0.055 | 7.962±0.028±0.088 |
| 3.8900 | 7.758±0.018±0.054 | 7.799±0.028±0.087 |
| 3.8950 | 7.567±0.018±0.053 | 7.626±0.027±0.085 |
| 3.9000 | 7.575±0.018±0.053 | 7.631±0.027±0.085 |
| 3.9050 | 7.596±0.018±0.053 | 7.625±0.027±0.085 |
| 3.9100 | 7.240±0.017±0.050 | 7.267±0.027±0.082 |
| 3.9150 | 7.454±0.018±0.052 | 7.533±0.027±0.088 |
| 3.9200 | 6.806±0.016±0.048 | 6.903±0.026±0.076 |
| 3.9250 | 6.694±0.016±0.046 | 6.763±0.026±0.075 |
| 3.9300 | 6.735±0.016±0.047 | 6.825±0.026±0.076 |
| 3.9350 | 7.161±0.017±0.051 | 7.144±0.027±0.079 |
| 3.9400 | 7.228±0.017±0.050 | 7.256±0.027±0.082 |
| 3.9450 | 7.590±0.018±0.054 | 7.608±0.028±0.086 |
| 3.9500 | 7.714±0.018±0.055 | 7.739±0.028±0.086 |
| 3.9550 | 8.124±0.019±0.056 | 8.141±0.029±0.090 |
| 3.9600 | 8.489±0.020±0.061 | 8.548±0.029±0.095 |
| 3.9650 | 7.768±0.018±0.054 | 7.770±0.028±0.086 |
| 3.9700 | 7.321±0.017±0.051 | 7.368±0.028±0.082 |
| 3.9750 | 8.062±0.019±0.057 | 8.050±0.029±0.089 |
| 3.9800 | 7.851±0.019±0.059 | 7.808±0.028±0.087 |
| 3.9850 | 7.969±0.019±0.057 | 7.992±0.029±0.089 |
| 3.9900 | 8.024±0.019±0.056 | 8.104±0.029±0.091 |
| 3.9950 | 7.985±0.019±0.057 | 7.984±0.028±0.084 |
| 4.0000 | 7.732±0.018±0.056 | 7.805±0.028±0.088 |
| 4.0050 | 7.537±0.018±0.053 | 7.567±0.028±0.085 |
| 4.0100 | 7.183±0.017±0.050 | 7.164±0.027±0.079 |
| 4.0120 | 6.907±0.017±0.051 | 6.951±0.027±0.079 |
| 4.0140 | 6.694±0.016±0.048 | 6.716±0.027±0.075 |
| 4.0160 | 6.544±0.016±0.045 | 6.582±0.026±0.074 |
| 4.0180 | 6.968±0.017±0.049 | 6.996±0.027±0.078 |
| 4.0200 | 6.726±0.016±0.047 | 6.735±0.027±0.075 |
| 4.0250 | 6.538±0.016±0.047 | 6.583±0.026±0.073 |
| 4.0300 | 16.451±0.036±0.115 | 16.526±0.042±0.187 |
| 4.0350 | 6.706±0.016±0.047 | 6.687±0.027±0.074 |
| 4.0400 | 6.564±0.016±0.046 | 6.640±0.027±0.073 |
| 4.0500 | 6.567±0.016±0.047 | 6.620±0.027±0.076 |
| 4.0550 | 6.927±0.017±0.052 | 6.934±0.027±0.077 |
| 4.0600 | 6.338±0.015±0.045 | 6.344±0.026±0.071 |

Continued

| $\sqrt{s}$/GeV | $e^+e^- \to (\gamma)e^+e^-$/pb$^{-1}$ | $e^+e^- \to (\gamma)\gamma\gamma$/pb$^{-1}$ |
|---|---|---|
| 4.0650 | 7.022±0.017±0.050 | 6.980±0.027±0.077 |
| 4.0700 | 7.271±0.017±0.052 | 7.292±0.028±0.079 |
| 4.0800 | 7.721±0.018±0.054 | 7.686±0.029±0.085 |
| 4.0900 | 7.611±0.018±0.054 | 7.647±0.029±0.084 |
| 4.1000 | 7.254±0.017±0.051 | 7.333±0.029±0.085 |
| 4.1100 | 7.146±0.017±0.050 | 7.219±0.028±0.080 |
| 4.1200 | 7.648±0.018±0.053 | 7.728±0.028±0.085 |
| 4.1300 | 7.207±0.017±0.051 | 7.187±0.029±0.079 |
| 4.1400 | 7.268±0.017±0.051 | 7.296±0.030±0.082 |
| 4.1450 | 7.774±0.019±0.057 | 7.837±0.029±0.092 |
| 4.1500 | 7.662±0.018±0.053 | 7.699±0.028±0.087 |
| 4.1600 | 7.954±0.019±0.056 | 7.982±0.030±0.090 |
| 4.1700 | 18.008±0.039±0.130 | 18.012±0.045±0.197 |
| 4.1800 | 7.309±0.018±0.051 | 7.366±0.029±0.082 |
| 4.1900 | 7.560±0.018±0.052 | 7.571±0.029±0.084 |
| 4.1950 | 7.503±0.018±0.054 | 7.535±0.029±0.084 |
| 4.2000 | 7.582±0.018±0.053 | 7.640±0.030±0.084 |
| 4.2030 | 6.815±0.017±0.048 | 6.838±0.028±0.080 |
| 4.2060 | 7.638±0.018±0.055 | 7.660±0.030±0.088 |
| 4.2100 | 7.678±0.018±0.054 | 7.764±0.030±0.089 |
| 4.2150 | 7.768±0.019±0.054 | 7.780±0.030±0.087 |
| 4.2200 | 7.935±0.019±0.055 | 7.963±0.030±0.088 |
| 4.2250 | 8.212±0.020±0.061 | 8.216±0.031±0.092 |
| 4.2300 | 8.193±0.020±0.057 | 8.249±0.031±0.093 |
| 4.2350 | 8.273±0.020±0.057 | 8.365±0.031±0.097 |
| 4.2400 | 7.830±0.019±0.054 | 7.858±0.030±0.087 |
| 4.2430 | 8.571±0.020±0.060 | 8.550±0.032±0.096 |
| 4.2450 | 8.487±0.020±0.060 | 8.523±0.032±0.095 |
| 4.2480 | 8.554±0.020±0.059 | 8.603±0.032±0.096 |
| 4.2500 | 8.596±0.020±0.060 | 8.599±0.032±0.095 |
| 4.2550 | 8.657±0.020±0.060 | 8.611±0.032±0.095 |
| 4.2600 | 8.880±0.021±0.063 | 8.905±0.032±0.099 |
| 4.2650 | 8.629±0.020±0.061 | 8.639±0.032±0.099 |

Continued

| $\sqrt{s}$/GeV | $e^+e^- \to (\gamma)e^+e^-$/pb$^{-1}$ | $e^+e^- \to (\gamma)\gamma\gamma$/pb$^{-1}$ |
|---|---|---|
| 4.2700 | 8.548±0.020±0.060 | 8.571±0.032±0.096 |
| 4.2750 | 8.567±0.020±0.060 | 8.571±0.032±0.099 |
| 4.2800 | 8.723±0.021±0.060 | 8.747±0.032±0.097 |
| 4.2850 | 8.596±0.020±0.059 | 8.627±0.032±0.097 |
| 4.2900 | 9.010±0.021±0.062 | 9.068±0.033±0.102 |
| 4.3000 | 8.453±0.020±0.064 | 8.456±0.031±0.095 |
| 4.3100 | 8.599±0.021±0.063 | 8.598±0.032±0.100 |
| 4.3200 | 9.342±0.022±0.065 | 9.336±0.033±0.109 |
| 4.3300 | 8.657±0.021±0.063 | 8.625±0.031±0.095 |
| 4.3400 | 8.700±0.021±0.061 | 8.680±0.031±0.097 |
| 4.3500 | 8.542±0.020±0.064 | 8.521±0.031±0.094 |
| 4.3600 | 8.063±0.019±0.057 | 8.084±0.031±0.090 |
| 4.3700 | 8.498±0.020±0.061 | 8.475±0.032±0.095 |
| 4.3800 | 8.158±0.020±0.060 | 8.189±0.031±0.092 |
| 4.3900 | 7.460±0.018±0.052 | 7.547±0.030±0.086 |
| 4.3950 | 7.430±0.018±0.052 | 7.364±0.030±0.083 |
| 4.4000 | 7.178±0.018±0.050 | 7.095±0.029±0.084 |
| 4.4100 | 6.352±0.016±0.045 | 6.390±0.028±0.071 |
| 4.4200 | 7.519±0.018±0.054 | 7.532±0.030±0.085 |
| 4.4250 | 7.436±0.018±0.052 | 7.443±0.030±0.083 |
| 4.4300 | 6.788±0.017±0.047 | 6.778±0.029±0.075 |
| 4.4400 | 7.634±0.019±0.053 | 7.622±0.030±0.087 |
| 4.4500 | 7.677±0.019±0.054 | 7.746±0.031±0.087 |
| 4.4600 | 8.724±0.021±0.072 | 8.731±0.033±0.101 |
| 4.4800 | 8.167±0.020±0.062 | 8.145±0.032±0.093 |
| 4.5000 | 7.997±0.019±0.056 | 7.954±0.032±0.088 |
| 4.5200 | 8.674±0.021±0.061 | 8.550±0.033±0.096 |
| 4.5400 | 9.335±0.022±0.077 | 9.263±0.034±0.102 |
| 4.5500 | 8.765±0.021±0.066 | 8.719±0.033±0.098 |
| 4.5600 | 8.259±0.020±0.068 | 8.117±0.032±0.090 |
| 4.5700 | 8.390±0.020±0.062 | 8.311±0.033±0.093 |
| 4.5800 | 8.545±0.021±0.060 | 8.491±0.033±0.094 |
| 4.5900 | 8.162±0.020±0.056 | 8.076±0.032±0.090 |

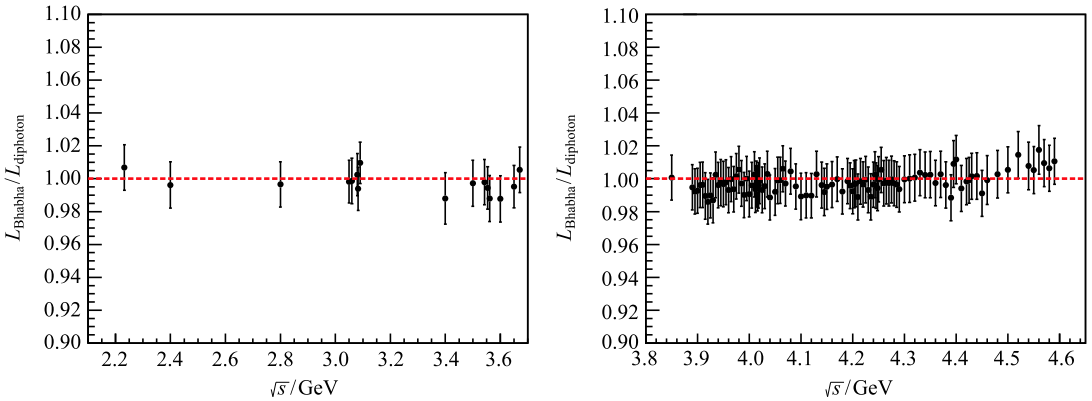


Fig. 3. (color online) The ratios of luminosities measured by $e^+e^- \to (\gamma)e^+e^-$ and $e^+e^- \to (\gamma)\gamma\gamma$. The right-hand plot is for the data samples with c.m. energy larger than 3.8500 GeV, while others are shown in the left-hand plot. The two methods give fully compatible results within the quoted uncertainties.

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