Predicting success on high-stakes math tests from preschool math measures among children from low-income homes

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Abstract

State mandated tests have taken center stage for assessing student learning and for holding teachers and students accountable for achieving adequate progress. What types of early knowledge predict performance on these tests, especially among low-income children who are at risk for poor performance? We report on a longitudinal study of 519 low-income American children from ages 5–12 with a focus on mathematics performance. We found that nonsymbolic quantity knowledge and repeating patterning knowledge at the end of preschool were reliable predictors of performance on standards-based high-stakes tests across three different grade levels (4^{th}–6^{th} grade), over and above other math and academic skills. Further, these effects of preschool math knowledge were partially mediated through symbolic mapping and calculation knowledge at the end of first grade. These findings suggest that nonsymbolic quantity knowledge and repeating patterning knowledge prior to formal schooling are valuable indicators of low-income children's performance on high-stakes state math tests in the middle grades.

KEYWORDS: Student Knowledge, Math Education, High Stakes Testing, Child Development, Longitudinal Studies

Educational Impact and Implications Statement

Students are typically required to complete annual, statewide tests that measure important learning outcomes and assess whether students meet state academic standards. We found that two skills in preschool were important predictors of how children performed on a high-stakes state math test seven years later: children's knowledge of repeating patterns (e.g., what two objects come next in this sequence?) and children's knowledge of nonsymbolic quantities (e.g., which picture shows more grapes?). These results can help inform early math content standards, which currently do not emphasize these two skills. Further, these results should encourage educators to attend to these two skills in early math settings as they may provide a foundation for later mathematics learning.

Predicting success on high-stakes math tests from preschool math measures among children from low-income homes

Low-income students often struggle to become proficient in mathematics. For example, on the 2015 National Assessment of Educational Progress (NAEP), only 24% of low-income fourth-grade students were at or above proficiency in math, and 28% were below basic (https://www.nationsreportcard.gov/). In contrast, 58% of students from more advantaged homes were at or above proficiency in math, and only 8% were below basic. Shin and colleagues (2013) report similar achievement gaps across fourth through seventh grade. One reason low-income students struggle in math is because they begin school with weaker math skills than their middle-income peers (Jordan, Kaplan, Olah, & Locuniak, 2006; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Starkey, Klein, & Wakeley, 2004). Further, one study suggests these differences in early math knowledge between lower- and middle-income students largely account for the differences in their later math achievement in third grade (Jordan et al., 2009).

The goal of the current paper is to expand our understanding of the types of early math knowledge that are important predictors of low-income children's performance on standards-based high-stakes tests in 4th through 6th grade. Prior research has focused on symbolic numeracy knowledge—knowledge of whole numbers and number relations, which includes skills like counting, calculation, and symbolic mapping (i.e., mapping between symbolic numerals, verbal number names, and their magnitudes; Purpura & Lonigan, 2013). Here, we test the hypothesis that two other skills before school entry—repeating patterning and nonsymbolic quantity knowledge—are predictors of low-income children's performance on a high-stakes state test many years later. Repeating patterning knowledge includes the ability to identify and extend predictable sequences that have a part that repeats (repeating patterns). Nonsymbolic quantity knowledge is knowledge of the magnitude of sets that does not rely on verbal or symbolic number names (e.g., a picture of four dots shows more than a picture of three dots). We also test the hypothesis that children's symbolic mapping, calculation and repeating patterning knowledge in first grade mediate this relation. We evaluated these hypotheses in a longitudinal sample of over 500 low-income children from ages 5-12. Below, we discuss the importance of predicting performance on high-stakes tests and then introduce the literature on early math knowledge.

High-Stakes Tests

In the current study, we focus on predicting performance on standards-based high-stakes tests in Tennessee. One of the cornerstones of The No Child Left Behind Act (NCLB, 2002) as well as the Every Student Succeeds Act (ESSA, 2015) is the requirement that students complete annual, statewide tests and demonstrate proficiency on state academic standards. As such, high-stakes state tests have taken center stage as indicators of student knowledge and as tools for holding schools, teachers, and students accountable for meeting the standards and achieving adequate yearly progress.

Most past research has focused on predicting mathematics knowledge using norm-referenced assessments administered by researchers. Although informative, performance on norm-referenced assessments may not generalize to performance on standards-based high-stakes tests. One reason is because the tests often serve different purposes (e.g., Bond, 1996; Linn, 2000). Norm-referenced tests are intended to discriminate between low- and high-achieving students and to compare student performance and growth over time to a reference group (the standardization sample; Betebenner, 2008). In contrast, standards-based or criterion-referenced tests are intended to measure whether students learn content irrespective of other students' performance. Theoretically, states could choose either type of assessment for high stakes outcomes, and there are suggestions for how to turn criterion-referenced measures into a “growth to standards” approach (Betebenner, 2008). However, like most states (Good, Wiley, & Sabers, 2010), Tennessee’s high-stakes tests are criterion-referenced to the state’s academic standards (https://www.tn.gov/education/instruction/academic-standards.html).

Another reason that performance may not generalize is that fluctuations in standards-based state test scores over time or between groups are not always mirrored in fluctuations in norm-referenced test scores (see Horn, 2003). Additionally, teachers often report that their instruction prioritizes the content and format of standards-based high-stakes state tests (Abrams, Pedulla, & Madaus, 2003), so the predictive power of early math skills may be attenuated relative to previous findings. One study reported a strong correlation (r = .77) between third-grade students’ scores on an individually-administered norm-referenced test (i.e., Woodcock-Johnson Assessment) and a standards-based state test (i.e., Delaware State Testing Program; Jordan et al., 2009). This suggests some support for the idea that performance may generalize across tests; however, given the differences between these test types, evidence is needed.

Critically, we have not found research using standards-based state tests that works to identify a range of early math skills that are predictive of later achievement. There is some limited research that suggests early symbolic numeracy knowledge predicts scores on future standards-based state tests (e.g., Jordan et al., 2010). For example, the Test of Early Numeracy (TEN-CBM) assesses kindergarten and first-grade students on four symbolic numeracy tasks, and it predicts performance on state tests at the end of third grade (Missall et al., 2012). However, little is known about predictors of state test scores beyond general symbolic numeracy knowledge or about predictors in preschool, when symbolic numeracy knowledge is limited.

Given that state tests are primary indicators of important learning outcomes, it is critical to identify predictors of performance on those tests in particular.

Preschool Math Knowledge

Based on a synthesis of the literature on early math cognition, Rittle-Johnson and colleagues (2017) identified three types of math knowledge in preschool that predict later mathematics knowledge on norm-referenced tests: counting, nonsymbolic quantity knowledge and repeating patterning knowledge. In preschool, there are sufficient individual differences in these three types of knowledge to predict later mathematics knowledge, although most studies have focused on only one type of early knowledge. Note that other types of math knowledge begin to develop in preschool, but individual differences in those knowledge types are just emerging at this age and have not been shown to predict later mathematics knowledge.

Among these early math skills, counting knowledge receives the most attention in preschool (Nguyen et al., 2016). Counting knowledge includes knowing the verbal number names, making a one-to-one correspondence between objects and verbal number names, and using the largest verbal number name to identify the cardinality of the set (Purpura & Lonigan, 2013). It draws on a range of cognitive skills, including linguistic, spatial, and quantitative skills (Ansari et al, 2003). Counting knowledge at ages four and five predicts performance on norm-referenced math tests in elementary school (Aunola, et al., 2004; Muldoon, Towse, Simms, Perra, & Menzies, 2013), including for children from low-income homes (Nguyen et al., 2016).

An earlier developing and more basic skill is now recognized as contributing to mathematics development: nonsymbolic quantity knowledge. Nonsymbolic quantity knowledge is knowledge of the magnitude of sets that does not rely on verbal or symbolic number names (e.g., a picture of four dots shows more than a picture of three dots, sometimes called quantitative skill). This knowledge begins to develop in infancy, including the ability to discriminate between small set sizes (Starkey & Cooper, 1980) as well as large set sizes (Xu, Spelke, & Goddard, 2005). Starting in preschool, individual differences in nonsymbolic quantity knowledge are related to performance on norm-referenced math tests six months to two years later (Feigenson, Libertus, & Halberda, 2013; LeFevre et al., 2010). The relation is strongest before age six (Fazio et al., 2014). Recently, we confirmed that these findings generalized to low-income children and to predicting math knowledge six years later on norm-referenced tests (Rittle-Johnson, Fyfe, Hofer, & Farran, 2017). Here, we tested the hypothesis that past findings would generalize to predicting standards-based high-stakes test scores five to seven years later.

Finally, repeating patterning knowledge is emerging as an important type of early math knowledge (see Burgoyne, Witteveen, Tolan, Malone, & Hulme, 2017). Repeating patterns are linear and have a unit that repeats (such as red-blue-blue, red-blue-blue). Four- and five-year-olds are often asked to copy or extend repeating patterns (Clarke, Clarke, & Cheeseman, 2006). Repeating patterning knowledge draws on multiple cognitive skills, including relational reasoning, executive function, and spatial skills (Miller, Rittle-Johnson, Loehr & Fyfe, 2016; Collins & Laski, 2015). Children who received an intervention in preschool focused primarily on repeating patterns had increased number knowledge at the end of kindergarten relative to children who had not received the intervention (Papic, Mulligan, & Mitchelmore, 2011). For example, children who received the patterning intervention were more successful counting forward and backward and knowing the number word before or after a given number word. Further, recent evidence indicates that low-income students' repeating patterning knowledge at the end of preschool was a unique predictor of fifth- and sixth-grade math knowledge on norm-referenced tests (Nguyen et al., 2016; Rittle-Johnson et al., 2017), in part because it predicted better numeracy knowledge in first grade. In this study, we tested the hypothesis that findings on the importance of early repeating patterning knowledge would generalize to predicting scores on standards-based high-stakes tests five to seven years later.

Compared to counting, repeating patterning and nonsymbolic quantity knowledge are likely less dependent on verbal skills and learned knowledge, such as the verbal number words and their meaning. Attention to regularities and to quantities in the environment should be less dependent on the verbal input of others and on the verbal skills of the child. This may be important among low-income children because they often receive less verbal input from adults and have poorer verbal skills than children from more affluent families (Hart & Risley, 2003).

Mediating Role of First-Grade Math Knowledge

A second goal of this paper was to work towards a better understanding of how repeating patterning and nonsymbolic quantity knowledge in preschool might support later performance on standards-based high-stakes tests. We explore the possibility that repeating patterning and nonsymbolic quantity knowledge provide foundational support for key skills in early elementary school, which in turn support later math achievement on high-stakes tests.

A recent synthesis of the early math cognition research indicated that, in early elementary school, symbolic mapping, calculation and patterning knowledge are three types of math knowledge that have been shown to predict later mathematics achievement (Rittle-Johnson et al., 2017). Symbolic mapping is knowledge of mappings between symbolic numerals, verbal number names, and magnitudes, and it includes a variety of skills such as labeling a set of objects with a verbal number name or a written numeral and comparing the values of verbal or written numbers (e.g., 7 is greater than 6; Hurst, Anderson, & Cordes, 2017; Jordan et al., 2006). There are well-established links between symbolic mapping knowledge in the primary grades and a range of math outcomes on norm-referenced tests across elementary school (De Smedt et al., 2013; Fazio et al., 2014; Fuchs et al., 2014; Geary & vanMarle, 2016).

Calculation knowledge is the ability to calculate the composition or decomposition of sets (Purpura & Lonigan, 2013). Calculating with the support of objects at ages five and six is correlated with performance on a norm-referenced test two years later (LeFevre et al., 2010). Further, general calculation knowledge in first and second grade is a strong predictor of performance on researcher-designed math measures and norm-referenced math achievement in the middle grades (Cowan et al., 2011; Geary, 2011; Jordan et al., 2013).

In addition to symbolic mapping and calculation, repeating patterning knowledge may continue to be a valuable predictor in early elementary school. Past research with elementary school children has typically focused on a broad range of pattern types, not just repeating patterns. They often describe patterns as knowledge of any predictable regularity, ranging from symmetrical patterns (A B C C B A) and growing patterns (2 4 6 8) to triangular patterns of dots and partitioning into halves, thirds and fourths (Mulligan & Mitchelmore, 2009; Kidd et al., 2014). Importantly, a six-month patterning intervention for struggling first-grade students improved their performance on a norm-referenced math test at the end of the school year, mediated through improvements in their broad patterning knowledge (Kidd et al., 2014; Pasnak, Kidd, Gadzichowski, Gallington, Schmerold & West, 2015). However, there is some concern that *repeating* patterns are not sufficiently advanced to support broader patterning knowledge, which often involves patterns in numbers and quantities (National Mathematics Advisory Panel, 2008). In contrast to this concern, recent evidence suggests that repeating patterning knowledge in first grade predicted later performance on norm-referenced math tests among low-income students (Rittle-Johnson et al., 2017). Thus, it is important to continue to evaluate the relevance of repeating patterning knowledge in early elementary school.

Nonsymbolic quantity knowledge and repeating patterning knowledge in preschool may help to develop these three skills (i.e., symbolic mapping, calculation, and repeating patterning) in first grade. For example, nonsymbolic quantity knowledge provides a foundation for mapping between magnitudes and verbal and symbolic numbers (Feigenson et al., 2013) as well as an intuitive understanding of basic calculations (Barth et al., 2005). Nonsymbolic knowledge of equivalent sets may also support copying a pattern or recognizing the number of elements in a single pattern unit. Improving repeating patterning knowledge in preschool has also been shown to support symbolic mapping and calculation knowledge (Papic et al., 2011). Working with repeating patterns may provide opportunities to deduce underlying rules, supporting future success detecting rules and regularities in numbers (e.g., Greenes, Ginsburg & Balfanz, 2004).

In sum, we predict that repeating patterning and nonsymbolic quantity knowledge in preschool would be unique predictors of students' performance on standards-based state math tests in 4^{th} through 6^{th} grade, over and above the influence of other math and academic skill. In addition, we predict that symbolic mapping, calculation, and possibly repeating patterning would be key predictors in first grade, and these first-grade skills would mediate the relations between preschool math knowledge and later math achievement. (Figure S1 in the supplemental materials provides a visual diagram of the predicted model). These relations are especially important to explore among low-income students whose later achievement is known to be problematic.

Method

Participants

Participants were from a longitudinal study of 771 children from low-income homes originally recruited at the beginning of prekindergarten in 2006 for a three-year study. Children were recruited from 57 pre-k classes at 20 public schools and 4 Head Start sites all of which served children who qualified for free or reduced priced lunch in a large urban city in Tennessee (family income less than 1.85 times the U.S. Federal income poverty guideline). Of the 771 children in the original sample, we located and re-consented 519 children in 2013 for a four-year follow-up study in middle school, all within the mid-state region. Based on available data, students in the final re-consented sample did not differ significantly from the students we were unable to locate or re-consent on key measures including sex, ethnicity, free and reduced priced lunch status, and early math scores (see Table S1 in the supplemental materials).

The final re-consented sample was 56% female, 79% black, 9% white, 8% Hispanic, 4% other races, and 9% English Learners. Based on maternal report when the children were in pre-k, 43% of families had an annual income under $10,000, 38% had an income between $10,000 and $25,000, and the remaining 19% had an income over $25,000; 25% of mothers had less than a high-school diploma, 33% had a high-school diploma or GED, and 42% had some post-secondary education. The same sample was used in Rittle-Johnson et al. (2017), but with an individually administered norm-referenced measure at a single grade level. The current analyses focused on two predictive time points: the end of pre-k (*M* age = 5.0) and the end of first grade (*M* age = 7.0). Analyses predicted outcome data at three later time points: fourth grade (*M* age = 10.1; 15% retained in third grade), fifth grade (*M* age = 11.1; 15% retained in fourth grade), and sixth grade (*M* age = 12.1; 17% retained in fifth grade).

Outcome Measure

The outcome measure for this study was from the Tennessee Comprehensive Assessment Program (TCAP), which served as Tennessee's standards-based high-stakes assessment for holding schools accountable through the period of this study. The TCAP assessment is group-administered by schools in a paper-and-pencil multiple-choice format. We obtained students' scaled scores on the *TCAP Math* subtest, which measures a range of skills, including the ability to estimate and compare whole numbers and fractions, solve algebraic expressions, identify shapes, and analyze data. Each item (n = 55) is from one of five categories: (1) Mathematical Processes, (2) Number and Operations, (3) Algebra, (4) Geometry and Measurement, and (5) Data Analysis, Statistics, and Probability. See Table 1 for the proportion of items in each of these categories across fourth, fifth, and sixth grade. In all three grades, the test heavily emphasized Numbers and Operations, with a gradual decrease in emphasis over grade levels. A similar trend occurred for items tapping Geometry and Measurement. In contrast, there was an increased emphasis on Algebra and Mathematical Processes across grade levels. Overall, there is strong evidence for the reliability of the assessment (e.g., KR-20 estimates for the Math assessment range from .92-.93) and validity of the assessment (e.g., correlation with ETS EXPLORE Math Test was .55; Tennessee Department of Education, 2015).

Predictor Measures

Early math knowledge. Math knowledge in pre-k and first grade was assessed using the Research-based Elementary Math Assessment (REMA). The REMA (Clements, Sarama, & Liu, 2008) has two parts, with one part focused on numeracy skills and the other part focused on geometry, repeating patterning and measurement skills. Based on our review of the literature, we broke the numeracy part into four subscales: nonsymbolic quantity, counting, symbolic mapping, and calculation. Purpura and Lonigan (2013) categorized REMA items similarly based on their confirmatory factor analyses, except they did not have a separate category for nonsymbolic quantity items. We also included the repeating patterning and shape subscales from the REMA; there were not enough measurement items given to create a measurement subscale. We included as many subscales as possible at each predictive time point to control for a variety of math skills and examine the unique contributions of our hypothesized predictors. Table 2 has example items and the number of items for each subscale (see also Table S2 and Table S3 in the supplemental materials). Below we report within-sample reliability estimates for each subscale using Cronbach's alpha values as well as omega total values (e.g., Revelle & Zinbarg, 2009).

Repeating Patterning. Repeating patterning items (α = .56-63, ω_t = .71-.80) tapped students' ability to identify and create predictable sequences. They involved working with repeating patterns made of colored shapes or blocks (e.g., extending a red-blue pattern by at least two blocks).

Nonsymbolic quantity. Nonsymbolic quantity items (α = 0.61-0.70, ω_t = .70-.86) tapped knowledge of the magnitude of set sizes from 2 to 12, without the need for verbal number labels or symbols. There was no time limit on these items, so children could count to help them complete the task, although counting was not encouraged. Thus, the nonsymbolic items did not directly assess the Approximate Number System or subitizing.

Counting. Counting items (α = 0.78-0.87, ω_t = .86-.88) primarily involved knowledge of the number-word sequence and counting sets of objects, though a few items involved identifying the cardinality of sets and detecting violations of the one-to-one correspondence principle. Most items involved quantities between 4 and 10, but a few items involved larger quantities (e.g., counting 30 pennies).

Symbolic Mapping. Symbolic mapping items (α = .77-.88, ω_t = .83-.90) involved mapping between symbolic numerals, their verbal number names and their magnitudes, including their relative magnitudes (e.g., matching the numeral 2 to a picture of two objects). Most items focused on numbers between 1 and 10, but a few items included two-digit numbers and one item included three-digit numbers.

Calculation. Calculation items (α = .91, ω_t = .92) involved combining or separating sets with or without objects (e.g., 4 chocolates plus 3 chocolates). Some calculation items were given at the end of pre-k, but performance was too low to use the measure at that time point. The majority of calculations were with numbers between 2 and 10, but some items involved two-digit numbers (e.g., 50 + 17).

Shape. Shape items (α = .89-96, ω_t = .97-99) focused on knowledge of shapes and their properties, including typical and atypical examples (e.g., picking out all the triangles from an array of shapes).

General academic and academic skills. We assessed four cognitive skills in pre-k and first grade to control for general academic skills in our models.

Reading ability. Reading ability is predictive of mathematics achievement (Duncan et al., 2007; Watts et al., 2014). The Woodcock Johnson III Letter-Word Identification subtest was used as a measure of early reading ability (see Duncan et al., 2007). It assesses children's abilities to identify isolated letters and words. The reported median reliability coefficient for this test is .94, using a split-half procedure (Schrank, McGrew, & Woodcock, 2001).

Narrative recall. Language skill, intelligence, and working memory capacity are also related to mathematics achievement (Duncan et al., 2007; Geary, 2011). Narrative recall was used as a direct measure of the combination of these skills, as evidence indicates that narrative recall is correlated with measures of vocabulary, verbal IQ and working memory (Florit et al., 2009). In pre-k, narrative recall was measured with the information score from the Renfrew Bus Story – North American Edition (Glasgow & Cowley, 1994), which scores the accuracy of children's retelling of a narrative. The Renfrew Bus Story developers indicate that the test-retest reliability of the information score is .79 (Hayward, Stewart, Phillips, Norris, & Lovell, 2008). In first grade, we used the Woodcock Johnson III Story Recall subtest, which has children answer questions about stories the assessor reads aloud. The reported median reliability coefficient for this test is .87 (Schrank, McGrew, & Woodcock, 2001).

Work-related skills. Attentive behavior, especially in the classroom, should increase children's opportunities to engage in and learn from instruction, and teacher-ratings of attentive behavior is predictive of later mathematics achievement (Duncan et al., 2007; Fuchs et al., 2014). Attentive behavior was measured via teacher ratings of children's work-related skills items on the Cooper-Farran Behavioral Rating Scale (Cooper & Farran, 1991). These 16 items assess children's attentiveness, ability to follow directions, and task persistence in the classroom on a 7-point behavior-anchored scale (α = .95-.96).

Self-regulated behavior. Self-regulated behavior includes the ability to plan and finish tasks, and self-regulation in the prekindergarten year is related to basic math knowledge at the end of prekindergarten and kindergarten (Blair & Razza, 2007; McClelland et al., 2007). Self-regulated behavior was measured via the Instrumental Competence Scale for Young Children-Short Form (Lange & Adler, 1997). Teachers rated behavior in the classroom on a 4-point Likert scale (α = .85-.89), and we used the four items focused on self-regulated behavior, such as "finishes tasks and activities."

Data Analysis

For early math predictors, missing data were rare and ranged from 0% to 7%. For early non-math predictors, the percent of missing cases for any given variable ranged from 0% to 22%, with most missing data involving teachers' ratings of work-related skills and self-regulated behavior. We used multiple imputation in SPSS to impute all missing values. Every analytic variable was included in the model. We imputed 30 datasets, which means the program output 30 unique datasets in which the missing values were replaced with plausible estimates. Resulting statistics were produced for each of the 30 unique datasets, and pooled results were produced that took into account variation across imputations. We report the pooled results.

In the middle grades, children were spread across different schools. To examine the amount of variability due to school, we calculated intraclass correlations on the outcome measures, which were moderate (*ICC*s = .16-.27). Because there was non-independence in the data, we used multi-level models with an individual level and a school level. We used the school level data from the year the TCAP scores were obtained. We treated the school intercept as a random effect. Individual predictors were mean centered at the individual level and treated as fixed effects. We used maximum likelihood (ML) to estimate the regression coefficients. We ran separate nested regression models for each predictive time point (end of pre-k and end of first grade) and for each outcome (TCAP scores in fourth, fifth, and sixth grade).

Results

Descriptive Statistics

Table 3 presents descriptive statistics for key variables and correlations between predictors and math outcomes. Table S4 in the supplemental material contains correlations among the predictors. All math predictors in pre-k and first grade were moderately correlated with scores on the TCAP Math assessment in the middle grades (*r*s = .27-.59). Patterns of correlations were similar across time.

Predicting TCAP Math Achievement

We examined whether knowledge of specific early math topics predicted performance on the TCAP Math assessment, after controlling for other math and cognitive skills and student demographics (see supplemental material for description of demographics). In each nested regression model, continuous variables were standardized so that parameter estimates represented standardized regression coefficients. Table 4 presents regression results from our fully controlled models. See Table S5 and Figure S1 in supplemental materials for details.

The results were consistent across outcome years and largely supported our hypotheses (see Table 4). First, we predicted that repeating patterning and nonsymbolic quantity knowledge in pre-k would be unique predictors of low-income children's performance on a high-stakes state test many years later. As shown in Table 4, at the end of pre-k, repeating patterning knowledge (βs = .14-.18) and nonsymbolic quantity knowledge (βs = .10-.14) were unique significant predictors of TCAP Math scores, and this was true across all outcome years assessed (fourth, fifth, and sixth grade). However, counting, shape, and symbolic mapping knowledge in pre-k were not unique predictors of math scores at any later grade.

Second, we hypothesized that symbolic mapping, calculation, and repeating patterning would be the key predictors in first grade. As shown in Table 4, at the end of first grade, symbolic mapping (βs = .15-.18) and calculation (βs = .19-.25) were strong, significant predictors of TCAP Math scores, and this was true across outcome years. Despite being a pre-k predictor, repeating patterning knowledge in first grade was not a unique predictor of TCAP Math scores (βs = .01-.06).

Third, we hypothesized that the key predictors in first grade would mediate the relations between pre-k math skills and later math achievement. Tables 5, 6, and 7 report mediation models for predicting fourth, fifth, and sixth-grade TCAP Math achievement respectively. We used nested regression models to complete the stepwise mediation approach recommended by Baron and Kenny (1986).

As shown in Tables 5, 6, and 7, the relevant predictors in pre-k were associated with the hypothesized mediators in first grade. For example, when predicting sixth-grade TCAP Math scores, the three unique predictors in pre-k were repeating patterning, nonsymbolic quantity, and narrative recall. Each of these three skills significantly predicted symbolic mapping and calculation in first grade (βs = .13-.32, see Table 5). Further, mediation results indicate that the direct associations between predictors at the end of pre-k and TCAP Math scores in fourth, fifth, and sixth grade were reduced in strength and often not significant after including the first-grade mediators. For example, the direct, significant associations between repeating patterning, nonsymbolic quantity, and narrative recall in pre-k and sixth-grade TCAP scores (βs = .14-.22) were reduced in strength and nonsignificant (βs = .08-.09) after including the first-grade mediators (Table 7). We used a supplemental bootstrapping technique recommended by Preacher and Hayes (2008), and the bootstrapping technique produced similar conclusions (see Table S7).

Recall that some students were retained a grade level and completed a different TCAP test (e.g., the third-grade TCAP instead of the fourth-grade TCAP). The results were similar when the retained children were dropped from the analysis (Table S6) with one noteworthy deviation: when retained children were dropped, symbolic mapping knowledge at the end of pre-k predicted fifth- and sixth-grade TCAP Math scores.

Discussion

Low-income students often begin school with weaker early math knowledge than their more advantaged peers (e.g., Jordan, et al., 2006; Starkey, Klein, & Wakeley, 2004), and increasing evidence suggests that early math knowledge is highly predictive of later math achievement (e.g., Duncan et al., 2007; Friso-van den Bos et al., 2015; Watts et al., 2014). The current study provides evidence for the importance of specific types of early math knowledge for predicting low-income students' performance on high-stakes math tests many years later. Specifically, nonsymbolic quantity and repeating patterning knowledge in preschool predicted scores on high-stakes math tests in fourth, fifth, and sixth grades after controlling for other math and non-math skills. By the end of first grade, symbolic mapping and calculation knowledge were the key predictors of later test performance. The results provide important evidence regarding key predictors of performance on standards-based high-stakes tests, tests that play a central role in assessing student learning and in holding students and teachers accountable for achieving success.

Recently, there has been increased attention to the learning and teaching of math knowledge in preschool (e.g., Piasta, Pelatti, & Lynnine, 2014). Yet, there is limited empirical evidence for which particular early math skills are linked to later achievement. Previous longitudinal research on representative samples has tended to use global measures of early math knowledge that incorporated a wide range of skills into a single score (e.g., Duncan et al., 2007; Watts et al., 2014). Other studies have examined specific skills, but they only included a single skill or numeracy measure, without controlling for other math skills (e.g., Jordan et al., 2009).

This past research has highlighted the role that early *numeracy* knowledge plays in supporting later mathematics achievement (e.g., Aunola et al., 2004; Jordan et al., 2009; Nguyen et al., 2016). Indeed, among early math skills, counting knowledge receives the most attention in preschool (Nguyen et al., 2016). Clements and Sarama (2007) identify counting as the "capstone of early numerical knowledge, and the necessary building blocks for all further work with number and operations" (p. 467). Much less research has focused on the supporting role of other early math skills – particularly near the end of preschool when basic counting skills are near mastery for many children. The current study helps fill that gap. Below we outline several contributions of the current research, and discuss limitations and future directions.

First, in line with other research (e.g., Papic et al., 2011; Rittle-Johnson et al., 2017), our findings suggest that repeating patterning plays a key role in early math understanding and should be integrated into theories of math development. Repeating patterning knowledge in preschool predicted symbolic mapping and calculation in first grade that in turn were related to success on high-stakes tests across fourth, fifth, and sixth grade. Repeating pattern knowledge draws on multiple cognitive skills, including relational reasoning and executive function (Miller et al., 2016). Working with repeating patterns in preschool may help children learn to extract underlying rules and regularities, thereby supporting future success in detecting regularities in symbolic mappings and calculations (e.g., any number plus one is the next number in the count sequence; Greenes, Ginsburg, & Balfanz, 2004).

Repeating patterning knowledge in first grade was not a unique predictor of success on the standards-based state test. One possibility is that knowledge of basic, *repeating* patterns (e.g., ABABAB) may be a less reliable predictor in first grade relative to more complex patterns (e.g., growing patterns). For example, a first-grade intervention on varied, complex patterns led to increased math scores at the end of the school year (Kidd et al., 2014). Indeed, there is some concern that repeating patterns are not sufficiently advanced to support broader patterning or quantity knowledge (National Mathematics Advisory Panel, 2008). However, some past research has found support for the predictive power of basic repeating patterning knowledge in first grade (Nguyen et al., 2016; Rittle-Johnson et al., 2017). Thus, more research is needed to consider the role of patterning in first grade, including multiple pattern types that vary in complexity.

Second, our findings are consistent with work on the importance of nonsymbolic knowledge in preschool (Feigenson et al., 2013; LeFevre et al., 2010) and on the contributions of symbolic mapping and calculation in primary school to later mathematics achievement (e.g., Geary, 2011). Further, the effect of nonsymbolic quantity knowledge in preschool was mediated by symbolic mapping and calculation knowledge in first grade (see also Price & Fuchs, 2016). This supports theories that suggest, with time, symbolic mapping knowledge replaces nonsymbolic quantity knowledge in predicting future math achievement (De Smedt et al., 2013). Indeed, nonsymbolic quantity knowledge is thought to support students' abilities to link quantities to the symbol system (Feigenson et al., 2013) and to gain an understanding of operating on quantities (Barth et al., 2005).

Third, in contrast to some prior research (e.g., Aunola, et al., 2004), counting knowledge at the end of preschool (age 5) was not a unique predictor of later achievement after controlling for a variety of math and non-math skills. One possibility is that our measure primarily captured basic counting knowledge, which may matter more at the beginning of preschool than at the end. For example, Nguyen and colleagues (2016) found that advanced counting knowledge at the end of preschool (e.g., identifying the cardinality of a set) was a unique predictor of fifth-grade math knowledge, but basic counting knowledge (e.g., maintaining one-to-one correspondence) was not. The researchers suggest that it is the more complex counting strategies that help develop children's later understanding of quantities and arithmetic (Nguyen et al., 2016). Another possibility is that basic counting knowledge may not make unique contributions over and above certain math skills. For example, past research on counting has not typically included measures of nonsymbolic quantity knowledge and repeating patterning knowledge. These latter two skills are likely less dependent on verbal input and learned knowledge (e.g., verbal number words and their meaning), which may be important among low-income children who often receive less verbal input from adults (Hart & Risley, 2003). In general, these results do not suggest that counting is unimportant; rather, it may be critical to consider its influence in light of other math skills and to distinguish between basic and advanced counting knowledge.

Practically, the current study may help inform math content standards and math instruction. The Common Core State Standards (2010) do not including repeating patterns as a math content standard at any grade level, which contrasts with national and past state math standards for preschool and kindergarten (e.g., National Association for the Education of Young Children, 2014). Nonsymbolic quantity knowledge also receives little attention in schools or in the Common Core State Standards (2010). Teachers, administrators and policy makers are particularly concerned with student performance on high-stakes state tests. Thus, evidence that repeating patterning and nonsymbolic quantity knowledge predict performance on a high-stakes state test may help convince stakeholders to pay more attention to both in early math settings.

Despite the contributions of this work, several limitations suggest directions for future research. For example, more research is needed to examine predictors of high-stakes tests with diverse sample from a range of economic backgrounds. For example, previous research has reported differences as a function of socio-economic status for repeating patterning knowledge (Klein & Starkey, 2004), but not for nonsymbolic quantity knowledge (Scalise, Daubert, & Ramani, 2017). Thus, it will be important to continue studying how these early skills matter for both low-income and middle-income children. Additionally, the number of studies using standards-based state math tests as the primary outcome measure is surprising low, and the current research is just a start. Predicting performance on standards-based high-stakes tests is critical given their prominence in education as well as their differences from norm referenced tests (e.g., Bond, 1996). Indeed, finding predictors of high-stakes test scores may be necessary to convince stakeholders of the potential importance of specific early math skills. Future research should help expand the current study by including different populations and age groups.

Also, several of our measures exhibited somewhat low reliability estimates, and it will be necessary to replicate these results using various analytic methods as well as measures that are operationalized in diverse ways. As one example, nonsymbolic quantity knowledge could be measured via Approximate Number System acuity or with tasks that systematically differentiate between small numbers (i.e., 1, 2, and 3) versus large numbers. Similarly, symbolic mapping knowledge could be measured using symbolic magnitude estimation (e.g., where does the number 78 go on a number line from 0 to 100). As another example, patterning knowledge could be expanded beyond repeating pattern knowledge to include a broader range of pattern types (e.g., growing patterns, symmetrical patterns; Mulligan & Mitchelmore, 2009), and counting knowledge could focus more directly on advanced skills like identifying the cardinality of a set (see Geary, vanMarle, Chu, Rouder, Hoard, & Nugent, 2018). In addition, there is not consistency in content across state tests, and thus generalizability of the findings to other high-stakes tests is needed. Finally, future research needs to provide experimental evidence to test whether each skill plays a causal role. There is some causal evidence that a broad patterning intervention improves math achievement (Kidd et al., 2014) and that a numeracy intervention improves calculation (Dyson et al., 2015), but these are just a start.

In sum, the current study helps identify early predictors of high-stakes tests among a low-income sample in a single state. Standards-based state tests have increased in importance as tools to evaluate students, teachers, and, schools. The knowledge students need to score well on these tests is not obtained solely in the middle grades when the tests occur. Rather, knowledge building follows a trajectory that begins in preschool. At the end of preschool, nonsymbolic quantity and repeating patterning knowledge predicted state math scores in fourth, fifth, and sixth grade. In first grade, symbolic mapping and calculation knowledge were the key predictors, and partially mediated the associations between the preschool predictors and later achievement. Given the widespread need to better understand the development and improvement of mathematics knowledge, it is imperative to attend to early math skills that predict performance on high-stakes tests, particularly for children who are at risk of academic failure.

References

Abrams, L. M., Pedulla, J. J., & Madaus, G. F. (2003). Views from the classroom: Teachers' opinions of statewide testing programs. *Theory Into Practice, 42*, 18-29. http://www.jstor.org/stable/1477315

Ansari, D., Donlan, C., Thomas, M., Ewing, S. A., Peen, T., & Karmiloff-Smith, A. (2003). What makes counting count? Verbal and visuo-spatial contributions to typical and atypical number development. *Journal of Experimental Child Psychology, 85*, 50-62. doi:10.1016/S0022-0965(03)00026-2

Aunola, K., Leskinen, E., Lerkkanen, M.-K., & Nurmi, J.-E. (2004). Developmental dynamics of math performance from preschool to grade 2. *Journal of Educational Psychology, 96*, 699-713. doi:10.1037/0022-0663.96.4.699

Baron, R. M., & Kenny, D. A. (1986). The moderator-mediator variable distinction in social psychology research: Conceptual, strategic, and statistical considerations. *Journal of Personality and Social Psychology, 51*, 1173-1182. doi:10.1037/0022-3514.51.6.1173

Barth, H., La Mont, K., Lipton, J., & Spelke, E. S. (2005). Abstract number and arithmetic in preschool children. *Proceedings of the National Academy of Sciences, 102*, 14116-14121. doi:10.1073/pnas.0505512102

Betebenner, D. (2008). *Norm- and criterion-referenced student growth*. Dover, NH: Center for Assessment. Retrieved from https://www.nciea.org/library/norm-and-criterion-referenced-student-growth.

Blair, C., & Razza, R. P. (2007). Relating effortful control, executive function, and false belief understanding to emering math and literacy ability in kindergarten. *Child Development, 78*, 647-663. doi:10.1111/j.1467-8624.2007.01019.x

Bond, L. A. (1996). Norm- and criterion-referenced testing. *Practical Assessment, Research, and Evaluation, 5*(2). Retrieved from http: http://pareonline.net/getvn.asp?v=5&n=2

Burgoyne, K., Witteveen, K., Tolan A., Malone, S., & Hulme, C. (2017). Pattern understanding: Relationships with arithmetic and reading development. *Child Development Perspectives, 11*, 239-244. doi:10.1111/cdep.12240

Clements, D. H., Sarama, J. H., & Liu, X. H. (2008). Development of a measure of early mathematics achievement using the rasch model: The research-based early maths assessment. *Educational Psychology*, *28*, 457-482. doi:10.1080/01443410701777272

Collins, M., & Laski, E. V. (2015). Preschoolers' strategies for solving visual pattern tasks. *Early Childhood Research Quarterly, 32*, 204-214. doi:10.1016/j.ecresq.2015.04.004

Cooper, D. H., & Farran, D. (1991). *The Cooper-Farran Behavioral Rating Scales*. Brandon, VT: Clinical Psychology Publishing.

De Smedt, B., Noël, M., Gilmore, C. K., & Ansari, D. (2013). The relationship between symbolic and nonsymbolic numerical magnitude processing and the typical and atypical development of mathematics: Evidence from brain and behavior. *Trends in Neuroscience and Education, 2*, 48-55. doi:10.1016/j.tine.2013.06.001

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., . . . Japel, C. (2007). School readiness and later achievement. *Developmental Psychology, 43*, 1428-1446. doi:10.1037/0012-1649.43.6.1428

Dyson, N., Jordan, N. C., Beliakoff, A., & Hassinger-Das, B. (2015). A kindergarten number-sense intervention with contrasting practice conditions for low-achieving children. *Journal for Research in Mathematics Education, 46*, 331-370. doi:10.5951/jresematheduc.46.3.0331

Fazio, L. K., Bailey, D. H., Thompson, C. A., & Siegler, R. S. (2014). Relations of different types of numerical magnitude representations to each other and to mathematics achievement. *Journal of Experimental Child Psychology, 123,* 53-72. doi:10.1016/j.jecp.2014.01.013

Feigenson, L., Libertus, M. E., & Halberda, J. (2013). Links between the intuitive sense of number and formal mathematics ability. *Child Development Perspectives, 7*, 74-79. doi:10.1111/cdep.12019

Florit, E., Roch, M., Altoè, G., & Levorato, M. C. (2009). Listening comprehension in preschoolers: The role of memory. *British Journal of Developmental Psychology, 27*, 935-951. doi:10.1348/026151008x397189

Friso-van den Bos, I., Kroesbergen, E. H., Van Lui, J. E. H., Xenidou-Dervou, I., Jonkman, L. M., Van der Schoot, M., & Van Lieshout, E. C. D. M. (2015). Longitudinal development of number line estimation and mathematics performance in primary school children. *Journal of Experimental Child Psycholoyg, 134,* 12-29. doi:10.1016/j.jecp.2015.02.002

Fuchs, L. S., Geary, D. C., Fuchs, D., Compton, D. L., & Hamlett, C. L. (2014). Sources of individual differences in emerging competence with numeration understanding versus multidigit calculation skill. *Journal of Educational Psychology, 106*, 482-498. doi:10.1037/a0034444

Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: A 5-year longitudinal study. *Developmental Psychology, 47*, 1539-1552. doi:10.1037/a0025510

Geary, D. C., & vanMarle, K. (2016). Young children’s core symbolic and nonsymbolic quantitative knowledge in the prediction of later mathematics achievement. *Developmental Psychology, 52,* 2130-2144. doi:10.1037/dev0000214

Geary, D. C., vanMarle, K., Chu, F. W., Rouder, J., Hoard, M. K., & Nugent, L. (2018). Early conceptuality understanding of cardinality predicts superior school-entry number-system knowledge. *Psychological Science, 29,* 191-205. doi:10.1177/0956797617729817

Glasgow, C., & Cowley, J. (1994). *Renfrew Bus Story test - North American Edition*. Centreville, DE: Centreville School.

Good, T., Wiley, C., & Sabers, D. (2010). Accountability and educational reform: A critical analysis of four perspectives and considerations for enhancing reform efforts. *Educational Psychologist, 45*, 138-148. doi:10.1080/00461521003720171

Greenes, C., Ginsburg, H. P., & Balfanz, R. (2004). Big math for little kids. *Early Childhood Research Quarterly, 19,* 159-166. doi:10.1016/j.ecresq.2004.01.010

Hart, B., & Risley, T. R. (2003). The early catastrophe: The 30 million word gap by age 3. *American Educator, 27*, 4-9.

Hayward, D. V., Stewart, G. E., Phillips, L. M., Norris, S. P., & Lovell, M. A. (2008). Introduction to the test reviews. Language, Phonological Awareness, and Reading Test Directory (pp. 1-10). Edmonton, AB: Canadian Centre for Research on Literacy. Retrieved from http://www.uofaweb.ualberta.ca/elementaryed/ccrl.cfm

Horn, C. (2003). High-stakes testing and students: Stopping or perpetuating the cycle of failure? *Theory Into Practice, 42,* 30-41. http://www.jstor.org/stable/1477316

Hurst, M., Anderson, U., & Cordes, S. (2017). The acquisition of mappings among number words, written numerals, and quantities in preschoolers. *Journal of Cognition and Development, 18,* 41-62. doi:10.1080/15248372.2016.1228653

Jordan, N. C., Hansen, N., Fuchs, L. S., Siegler, R. S., Gersten, R., & Micklos, D. (2013). Developmental predictors of fraction conceps and procedures. *Journal of Experimental Child Psychology, 116,* 45-58. doi: 10.1016/j.jecp.2013.02.001

Jordan, N. C., Kaplan, D., Nabors Olah, L., & Locuniak, M. N. (2006). Number sense growth in kindergarten: A longitudinal investigation of children at risk for mathematics difficulties. *Child Development, 77*, 153-175. doi:10.1111/j.1467-8624.2006.00862.x

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: kindergarten number competence and later mathematics outcomes. *Developmental Psychology, 45*, 850-867. doi:10.1037/a0014939

Kidd, J. K., Pasnak, R., Gadzichowski, K. M., Gallington, D. A., McKnight, P., Boyer, C. E., & Carlson, A. (2014). Instructing first-grade children on patterning improves reading and mathematics. *Early Education & Development, 25*, 134-151. doi:10.1080/10409289.2013.794448

Klein, A., & Starkey, P. (2004). Fostering preschool children's mathematical knowledge: Findings from the Berkeley Math Readiness Project. In Douglas Clements & Julie Sarama (Eds.,) *Engaging young children in mathematics: Stanfards for early childhood mathematics education.* Mahwah, NJ: Lawrence Erlbaum Associates.

Lange, G., & Adler, F. (1997). *Motivation and achievement in elementary children*. Paper presented at the Bienniel meeting of the Society for Research in Child Development, Washington, DC.

LeFevre, J. A., Fast, L., Skwarchuk, S. L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of performance. *Child Development, 81*, 1753-1767. doi:10.1111/j.1467-8624.2010.01508.x

Linn. R. L. (2000). Assessments and accountability. *Educational Researcher, 29,* 4-16. doi:10.3102/0013189x029002004

McClelland, M. M., Cameron, C. E., Connor, C. M., Farris, C. L., Jewkes, A. M., & Morrison, F. J. (2007). Links between behavioral regulation and preschoolers' literacy, vocabulary, and math skills. *Developmental Psychology, 43*, 947-959. doi:10.1037/0012-1649.43.4.947

Miller, M. R., Rittle-Johnson, B., Loehr, A. L., & Fyfe, E. R. (2016). The influence of relational knowledge and executive function on preschoolers' repeating pattern knowledge. *Journal of Cognition and Development, 17,* 85-104. doi:10.1080/15248372.2015.1023307

Muldoon, K. P., Towse, J., Simms, V., Perra, O., & Menzies, V. (2013). A longitudinal analysis of estimation, counting skills, and mathematical ability across the first school year. *Developmental Psychology, 49*, 250-257. doi:10.1037/a0028240

Mulligan, J., & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal, 21*, 33-49. doi:10.1007/BF03217544

National Association for the Education of Young Children. (2014). NAEYC early childhood program standards and accreditation criteria. Washington DC: Authors.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Washington, DC: Authors.

National Mathematics Advisory Panel. (2008). *Foundations of success: The final report of the National Mathematics Advisory Panel.* Washington, DC: U.S. Department of Education.

Nguyen, T., Watts, T. W., Duncan, G. J., Clements, D. H., Sarama, J. S., Wolfe, C., & Spitler, M. E. (2016). Which preschool mathematics competencies are most predictive of fifth grade achievement? *Early Childhood Research Quarterly, 36,* 550-560. doi: 10.1016/j.ecresq.2016.02.003

Papic, M. M., Mulligan, J. T., & Mitchelmore, M. C. (2011). Assessing the development of preschoolers' mathematical patterning. *Journal for Research in Mathematics Education*, *42*, 237-268.

Pasnak, R., Kidd, J. K., Gadzichowski, K. M., Gallington, D. A., Schmerold, K. L., & West, H. (2015). Abstracting sequences: Reasoning that is a key to academic achievement. *The Journal of Genetic Psychology, 176,* 171-193. doi:10.1080/00221325.2015.1024198

Piasta, S., Pelatti, C., & Lynnine, H. (2014). Mathematics and Science Learning Opportunities in Preschool Classrooms. *Early Education and Development*, *25*(4), 445-468. doi:10.1080/10409289.2013.817753

Preacher, K. J., & Hayes, A. F. (2008). Asymptotic and resampling strategies for assessing and comparing indirect effects in multiple mediator models. *Behavior Research Methods, 40*, 879-891. doi:10.3758/brm.40.3.879

Price, G. R., & Fuchs, L. S. (2016). The mediating relation between symbolic and nonsymbolic foundations of math competence. *PLoS ONE*, *11*, e0148981. doi:10.1371/journal.pone.0148981

Purpura, D. J., & Lonigan, C. J. (2013). Informal numeracy skills: The structure and relations among numbering, relations, and arithmetic operations in preschool. *American Educational Research Journal, 50,* 178-209. doi:10.3102/0002831212465332

Revelle, W., & Zinbarg, R. E. (2009). Coefficients alpha, beta, omega, and the glb: Comments on Sijtsma. *Psychometrika, 74*, 145-154. Doi:10.1007/s11336-008-9102-z

Rittle-Johnson, B., Fyfe, E. R., Hofer, K. G., & Farran, D. C. (2017). Early math trajectories: Low-income children's mathematics knowledge from ages 4 to 11. *Child Development, 88*, 1727-1742. doi:10.1111/cdev.12662

Scalise, N., Daubert, E., & Ramani, G. B. (2017). Narrowing the early mathematics gap: A play-based intervention to promote low-income preschoolers' number skills. *Journal of Numerical Cognition, 3,* 559-581. doi:10.5964/jnc.v3i3.72

Schrank, F. A., McGrew, K. S., & Woodcock, R. W. (2001). *Technical Abstract* (Woodcock-Johnson III Assessment Service Bulletin No. 2). Itasca, IL: Riverside Publishing. Retrieved from http://www.hmhco.com/~/media/sites/home/hmh-assessments/clinical/woodcock-johnson/pdf/wjiii/wjiii_asb2.pdf?la=en

Starkey, P., & Cooper, R. G. (1980). Perception of numbers by human infants. *Science, 210*, 1033-1035. doi:10.1126/science.7434014

Tennessee Department of Education (2015). Technical report for the Tennessee Comprehensive Assessment Program Achievement Test. Upper Saddle River, NJ: Pearson.

Watts, T. W., Duncan, G. J., Siegler, R. S., & Davis-Kean, P. E. (2014). What's past is prologue: Relations between early mathematics knowledge and high school achievement. *Educational Researcher, 43,* 352-360. doi:10.3102/0013189x14553660

Xu, F., Spelke, E. S., & Goddard, S. (2005). Number sense in human infants. *Developmental Science, 8*, 88-101. doi:10.1111/j.1467-7687.2005.00395.x

Table 1

Proportion of Items in each Category on the TCAP MATH Subtest.

	Fourth Grade Test		**Fifth Grade Test**		**Sixth Grade Test**	
	N Items	Proportion	N Items	Proportion	N Items	Proportion
Mathematical Processes	6	.11	7	.13	14	.25
Number and Operations	25	.45	22	.40	20	.36
Algebra	6	.11	11	.20	11	.20
Geometry and Measurement	13	.24	10	.18	5	.9
Data Analysis, Statistics, and Probability	5	.9	5	.9	5	.9
TOTAL	55	1.00	55	1.00	55	1.00

Note. This information is based on the TCAP Math tests administered in Spring 2015, which is the year that most students in the current sample were in sixth grade (Tennessee Department of Education, 2015).

Table 2

Example Items and Number of Items for each Early Math Subscale

Subscale	Example Easier Item	Example Harder Item	Number of Items	
			End of Pre-K	End of First
Repeating Patterning	Identify the missing element in the pattern ABA_AB.	Duplicate a single copy of the core unit of a pattern.	6	7
Nonsymbolic Quantity	Shown two cards, with 4 dots and 3 dots: “Which one has more?”	Put connected cube towers in order from smallest to largest (towers made of 6-12 cubes).	7	9
Counting	Shown five objects in a line: “I bought these cans of food. Count these cans. Tell me how many there are.”	Count 30 pennies and identify how many there are.	22	25
Symbolic Mapping	Match the numerals 1-5 to the appropriate number of grapes.	Asked to compare: “Which is smaller, 27 or 32?”	15	18
Calculation	“Here are six pennies. Three more are hidden under the cloth. How many are there in all?”	Add 3 more to 69.	--	42
Shape	Select all triangles from a collection of 26 shapes; some are prototypic and some are not.	Fill 6 outlines of regular hexagons with simple shapes, using different compositions.	14	23

Note. The hardest items on each subscale were not given at the earlier time point. For the shape scale, the number of individually *scored* items was higher than the number of administered items (114 at the end of pre-k and 123 at the end of first grade). On four items, children were presented with a map of 26 shapes and they were asked to select all the examples of a certain shape (e.g., triangles). Each of the 26 shapes was scored, and the final score on each of the four items represented a weighted sum of the 26 items (e.g., getting credit for selecting correct shapes and for not selecting incorrect shapes, see Clements & Sarama, 2008).

Table 3

Descriptive Statistics and Correlations for Key Outcomes and Predictors

	Raw Score	Correlation with TCAP Score		
	M (SD)	1	2	3
Outcome Measures				
(1) Fourth-grade TCAP Math	739.30 (37.94)	--		
(2) Fifth-grade TCAP Math	750.16 (40.80)	.64	--	
(3) Sixth-grade TCAP Math	746.41 (40.21)	.60	.68	--
Math Predictors				
Repeating Patterning (pre-k)	1.97 (1.39)	.42	.47	.39
Repeating Patterning (first grade)	3.67 (1.09)	.27	.33	.32
Nonsymbolic quantity (pre-k)	3.78 (1.51)	.39	.43	.37
Nonsymbolic quantity (first grade)	7.99 (1.34)	.44	.57	.46
Counting (pre-k)	8.61 (4.34)	.41	.46	.38
Counting (first grade)	18.91 (2.48)	.42	.52	.42
Symbolic mapping (pre-k)	5.40 (3.27)	.42	.47	.39
Symbolic mapping (first grade)	13.94 (2.34)	.49	.59	.50
Calculation (first grade)	9.55 (6.23)	.52	.58	.50
Shape (pre-k)	12.36 (3.45)	.33	.38	.31
Shape (first grade)	15.75 (2.90)	.32	.35	.31
Non-Math Predictors				
Reading ability (pre-k)	347.64 (21.69)	.35	.34	.27
Reading ability (first grade)	437.83 (26.36)	.39	.48	.37
Narrative recall (pre-k)	13.79 (7.31)	.30	.29	.287
Narrative recall (first grade)	493.37 (6.69)	.23	.24	.24
Work-related skills (pre-k)	4.97 (1.26)	.32	.40	.33
Work-related skills (first grade)	4.89 (1.19)	.41	.53	.45
Self-regulation (pre-k)	2.84 (0.70)	.29	.38	.32
Self-regulation (first grade)	2.70 (0.69)	.37	.49	.44

Note. All correlations presented are statistically significant at $p < .05$. TCAP = Tennessee Comprehensive Assessment Program. For conciseness, we labeled each TCAP assessment based on the grade level of the majority of students, but some students were retained and completed the version for the previous grade level.

Table 4

Regression Estimates Predicting TCAP MATH Scores from Early Math and Non-Math Skills (N = 519)

	Pre-K Measures Predicting TCAP MATH			First Grade Measures Predicting TCAP MATH		
Measure	4th grade	5th grade	6th grade	4th grade	5th grade	6th grade
Math Predictors						
Repeating Patterning	.18**	.18**	.14*	.01	.02	.06
	(.06)	(.06)	(.06)	(.05)	(.05)	(.05)
Nonsymbolic Quantity	.14*	.10*	.13*	.03	.11	.10
	(.06)	(.05)	(.06)	(.06)	(.06)	(.06)
Counting	.04	.02	.01	.03	.04	.00
	(.08)	(.07)	(.07)	(.06)	(.06)	(.06)
Symbolic Mapping	.11	.11	.11	.15*	.18**	.16*
	(.07)	(.07)	(.07)	(.07)	(.06)	(.06)
Calculation	--	--	--	.25***	.19***	.21***
	--	--	--	(.06)	(.05)	(.06)
Shape	-.02	.01	-.00	.05	-.00	-.03
	(.06)	(.05)	(.06)	(.05)	(.05)	(.05)
Non-Math Predictors						
Reading Ability	.15**	.06	.06	.11*	.11*	.09
	(.05)	(.05)	(.05)	(.05)	(.05)	(.05)
Narrative Recall	.11*	.09*	.10*	.00	-.03	.03
	(.05)	(.04)	(.05)	(.04)	(.04)	(.04)
Work-Related Skills	.07	.04	.05	.22***	.22**	.15*
	(.08)	(.07)	(.08)	(.08)	(.07)	(.07)
Self-regulation	.02	.09	.10	-.05	-.02	.07
	(.07)	(.07)	(.07)	(.08)	(.07)	(.07)
Controls	Included	Included	Included	Included	Included	Included

Note. Standard errors are in parentheses. Included = all models presented include the full list of control variables: age at predictor time point, retained a grade level, gender, ELL status in pre-k, ethnicity, pre-k school type (public or Head Start), and socio-economic status (respondent's education level and income level). *$p < .05$. **$p < .01$. ***$p < .001$.

Table 5

First-Grade Skills as Mediators of the Association Between End of Pre-K Skills and Fourth-Grade TCAP MATH Scores

	Predicting Mediators in First Grade	
	Symbolic Mapping	Calculation
Pre-k predictors		
Repeating Patterning	.20***	.30***
Nonsymbolic quantity	.22***	.21***
Reading ability	.18***	.11**
Narrative recall	.10*	.11**
Controls	Included	Included

	Predicting Fourth-Grade TCAP MATH	
	Without Mediators	With Mediators
Pre-k predictors		
Repeating Patterning	.22***	.11*
Nonsymbolic quantity	.21***	.11*
Reading ability	.19***	.13**
Narrative recall	.12**	.07
First-grade mediators		
Symbolic mapping	--	.19**
Calculation	--	.25***
Controls	Included	Included

Note. Included = all models presented include the full list of control variables: age at end of pre-k, retained a grade level, gender, ELL status in pre-k, ethnicity, pre-k school type (Head Start or public), and socio-economic status (respondent's education level and income level). *$p < .05$. **$p < .01$. ***$p < .001$.

Table 6

First-Grade Skills as Mediators of the Association Between End of Pre-K Skills and Fifth-Grade TCAP MATH Scores

	Predicting Mediators in First Grade	
	Symbolic Mapping	Calculation
Pre-k predictors		
Repeating Patterning	.24***	.32***
Nonsymbolic quantity	.24***	.22***
Narrative recall	.13**	.13**
Controls	Included	Included

	Predicting Fifth-Grade TCAP MATH	
	Without Mediators	With Mediators
Pre-k predictors		
Repeating Patterning	.27***	.13**
Nonsymbolic quantity	.18***	.08
Narrative recall	.13**	.06
First-grade mediators		
Symbolic mapping	--	.28***
Calculation	--	.23***
Controls	Included	Included

Note. Included = all models presented include the full list of control variables: age at end of pre-k, retained a grade level, gender, ELL status in pre-k, ethnicity, pre-k school type (Head Start or public), and socio-economic status (respondent's education level and income level). $^*p < .05$. $^{**}p < .01$. $^{***}p < .001$.

Table 7

First-Grade Skills as Mediators of the Association Between End of Pre-K Skills and Sixth-Grade TCAP MATH Scores

	Predicting Mediators in First Grade	
	Symbolic Mapping	Calculation
Pre-k predictors		
Repeating Patterning	.24***	.32***
Nonsymbolic quantity	.24***	.22***
Narrative recall	.13**	.13**
Controls	Included	Included
	Predicting Sixth-Grade TCAP MATH	
	Without Mediators	With Mediators
Pre-k predictors		
Repeating Patterning	.22***	.08
Nonsymbolic quantity	.21***	.09
Narrative recall	.14**	.08
First-grade mediators		
Symbolic mapping	--	.25***
Calculation	--	.26***
Controls	Included	Included

Note. Included = all models presented include the full list of control variables: age at end of pre-k, retained a grade level, gender, ELL status in pre-k, ethnicity, pre-k school type (Head Start or public), and socio-economic status (respondent's education level and income level). $*p < .05$. $**p < .01$. $***p < .001$